VECTORS IN THREE DIMENSIONS

CONTENTS

(a) Scalars Product of Vectors in Three Dimensions
(b) Application of Scalar Product

SUB TOPIC: SCALAR PRODUCT OF VECTORS IN THREE DIMENSIONS

The scalar product of three vectors a,b and c is defined as a.(bxc) which is a scalar quantity.

If three vectors a,b and c are given as:

$a = a_1i + a_2j + a_3k$

$b = b_1i + b_2j + b_3k$

$c = c_1i + c_2j + c_3k$

The scalar product is found the same way as the determinant of a 3x3 matrix.

We denote dot or scalar product of two vectors A and B by A.B. This dot product is defined as the product of the magnitudes of A and B and the cosine of the angle between them.

A.B =

Scalar product otherwise called dot product (or inner products).

The Scalar product between two perpendicular vectors is zero.

Summary of the scalar product of unit vectors is provided below:

i.i = j.j = k.k = 1 here, $\theta = 0^0$

i.j = i.k = j.k = 0 here, $\theta = 90^0$

scalar product is scalar and commutative

A.B = B.A

Note:

a.(bxc) =(axb).c
Three vectors a,b and c are said to be coplanar or collinear if their scalar triple product is zero.

Properties of dot product

A.B = B.A

A.(B+C) = A.B +A.C
m(A.B) =(mA).B = (A.B)m where m is a scalar.
i.i = j.j = k.k = 1
i.j = j.k = k.i = 0

If A = a_1i + a_2j + a_3k and B = b_1i + b_2j + b_3k

A.B = $a_1b_1 + a_2b_2 + a_3b_3$

A.A = $a_1^2 + b_2^2 + b_3^2$

B.B = $b_1^2 + b_2^2 + b_3^2$

If A.B = 0 and A and B are not null vectors, then A and B are perpendicular.

Examples

Given: a = 2i –j + k; b = 3i + 2j – k ; c = i -4j + 3k
show that vectors a,b and c are coplanar.

Solution:

We need to show that a.(bxc) = 0

=2(6-4)+1(9+1)+ (-12-2) = 2 x2 + 10 – 14 = 14 – 14 = 0

Since a.(bxc) = 0, vectors a, b and c are coplanar.

If p = 2i + 5j – 3k
q = i + 0j + 5k
r = 3i – 4j + 2k, show if they are collinear or not.

Solution:

= 2(0+20) - 5 (2-15) -3 (-4-0) = 2 x20 – 5x – 13 -3x-4

= 40 + 65 + 12 = 117

Since

CLASS ACTIVITY:

Given a = i+2j+-3k, b = 2i – j + 2k, c = 3i + j – k. Find a.(bxc) = 0
P, q and r are three vectors given by 4i – j + 2k, 3i + 2j – 5k and –i+3j + k respectively. Evaluate (pxq).r

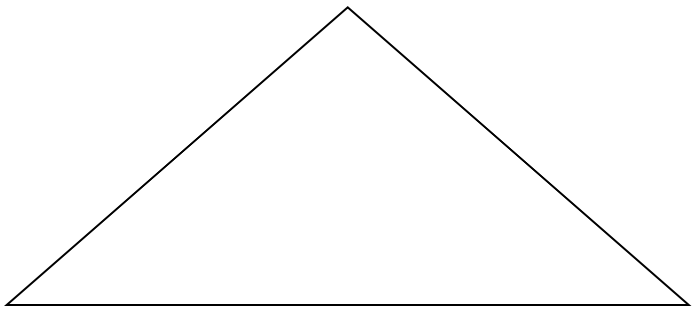
SUB TOPIC: APPLICATION OF SCALAR PRODUCT

We shall begin by applying the scalar product to the lines that form the sides of a triangle.

The application shall lead us to establish the cosine rule and the famous pythagoras' theorem.

Let

Applying triangle law of vectors



. = (

c^2 = - 2.

$c^2 = a^2+b^2 - 2ab\cos\theta$

The formula via, $c^2 = a^2+b^2 - 2ab\cos\theta$ is the familiar cosine rule.

If the vectors are perpendicular, $c^2 = a^2+b^2$

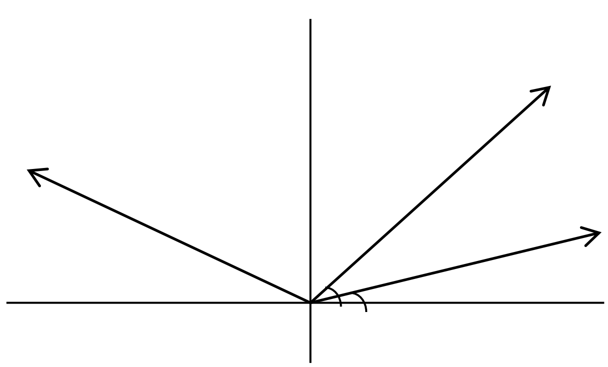
 = 0 and the formula will reduce to $c^2 = a^2+b^{2=}$

This is the well known Pythagoras's theorem for a right angled triangle.

Application of dot product to obtain trigonometric expansion.

Let O_x and O_y be perpendicular axes with unit vectors directions OA and OB which are perpendicular to one another. Let OA make angle x^0 with Ox.

Let unit vector act in the direction OC which makes angle y^0 with OA.



Resolving along Ox and and Oy gives

Resolving along Ox and Oy gives

= -isinx + cosx

Resolving along OA and OB gives

=

Substituting for

=

... (i)

Resolving along Ox and Oy gives

= ... (ii)

Since (i) and (ii) represent the same vector, corresponding components must be equal and so we have

Cos(x+y) = cos x cosy – sinxsiny

Sin(x+y) = sin x cosy + cosxsiny

CLASS ACTIVITY:

If

Prove the scalar product to the lines that form the sides of a triangle.

PRACTICE EXERCISE

Define the scalars product of vectors in three dimension.
Scalar product otherwise called ____ product
List all the properties of dot product.
Prove the applications of dot product to obtain trigonometric expansion.
If = 2i + 3j-k, = -i + 2j-4k, find

Prove that x = i +2j-3k,

ASSIGNMENT

Given that a = 3i + 2j + k and b = 4i – 5j +3k. find a.(axb); (b) b. (axb)
Show if question 2 is collinear.

The vertices of a triangle have position vectors 3i – 2j + 7k, 2i + 4j +k and 5i + 3j – 2k. find the area of the triangle.
List three properties of scalar product
Prove the application of dot product to obtain trigonometric expansion.

VECTORS IN THREE DIMENSIONS

CONTENTS

(a) Vectors or cross product in three dimensions and Properties of vector product.

(b) Application of cross product.

SUB TOPIC: VECTOR OR CROSS PRODUCT AND PROPERTIES

If a, b and c are three vectors, then ax(bxc) and (axb)xc are called the vector.

Note: ax(bxc) ≠ (axb)xc

Vector product is called cross product or outer product. It produces vector.

Properties of Vector Product

thus the vector product of two vectors is not commutative.

If a x b = 0 and a and b are non zero vectors, then a and b are parallel.
If

Examples:
Find the vector product of a and b where: a =4i – 3j + k and b = -I + 2j + 3k
Hence find |a x b| of the question above.
Solution

|a x b| = |

CLASS ACTIVITY:

What do we mean by vector or cross product
______ vectors are vectors on the same plane

Ax(BxC) is _____ combination of B and C.
If a =3i – 5j + 2k and b = 4i – 5j +3k, find a x b and |axb|
List three properties of cross vector.

SUB TOPIC: APPLICATIONS OF CROSS PRODUCT

Vector product as volume

Vector products can be used to determine the volume of a parallelepiped. If three vectors a, b, c represent the length, width and height of a parallelepiped, the volume of the parallelepiped is given by the scalar triple product of a,b,c.

V = (axb).c

Examples

Find the volume of a parallelepiped whose vectors which are the sides given a = 2i + j + k, b = i -3j +2k, c=3i+2j-k.

Solution

Volume of the parallelepiped is given as

V = (axb).c

But (axb).c = a.(bxc)

a.(bxc) = =2 = 2(3-4) – (-1-6)+ (2+9)

= - 2 + 7 + 11 = 16 units

Find the volume of parallelepiped whose vectors which sides are given as a = 3i + j + 2k, b = i -2j + 3k, c = 4i + j – 2k

Solution: Volume (axb).c

But (axb).c = a.(bxc)

a.(bxc) = = 3

= 3(4-3) – (-2-12) + (1+9) = 3+14+18= 35 units

CLASS ACTIVITY

If Ã = 3i-j+2k, = 2i +j – k and = i – 2j + 2k. Find (a) (Ãx) x (b) Ãx x)
Find the volume of a parallelepiped whose vectors which are the sides given x = 2i +j+k, y=i-3j+2k, z = 3i + 2j –k

PRACTICE EXERCISE

a,b,c are three vectors vectors, a = -6i+2j+k, b= 3i-2j+4k, c=5i+7j+3k. Find ax(bxc).
If a = 2i+2j+3k, b =-1+2j+k and c=3i+j. Find (axb)xc.
Define vector or cross product

Coplanar _____ is linear combination B and C
Find the sine of the angle between the vectors, p = I + j + k and q = 8i + 2j + 3k.
Three points A, B and C exist with AB=2i+j and BC=2i+5j. Find angle A B C.