DYNAMICS

CONTENT

(a) Newton's Laws of Motion

(b) Motion along Inclined Plane.

(c) Motion of Connected Particles

SUB TOPIC: NEWTON'S LAW OF MOTION

PARTICLE: This is a body or thing that has a negligible dimension. It is usually denoted by a point or dot.

MASS OF A BODY: This is the amount of matter contained in a body. It is measured in Kilogram me (Kg).

LINEAR MOMENTUM: The linear momentum of a particle is the product of the mass of the particle and its velocity. It is also known as momentum.

If mass= m, velocity= v and momentum= p.

Then, p=mv

NEWTON'S LAWS

FIRST LAW: Everybody or object remains at rest or of uniform motion unless compelled by an external force to act otherwise.

SECOND LAW: The rate of change of momentum of a body is proportional to the applied force and is in the direction of the force. (F =ma)

If a force F acts on a body of a mass m (kg) it produces acceleration (a) in the mass given by the relation. F=ma

THIRD LAW: To every action, there is always an opposite and equal reaction.

Examples:

A force P acts on a body of mass 5kg on a smooth horizontal floor. If it produces an acceleration of $4.5ms^{-2}$, find the magnitude of P.

Solution: $a=4.5ms^{-2} = 4\ ½\ ms^{-2} = 9/2\ ms^{-2}$.

M= 5kg.

P= ma = 5 X 9/2 = 22.5N

A body of mass 100kg is placed in a lift. find the reaction between the floor of the lift and the body when the lift moves upward

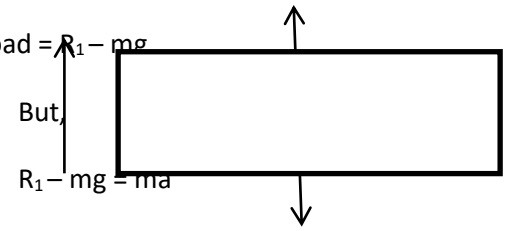
At constant velocity

With an acceleration of $3.5ms^{-2}$ ($g=10ms^{-2}$)

Solution

Let R_1 be the reaction between the floor of the lift and the load when the lift moves upward at constant velocity

The net force on the load = $R_1 - mg$



But,

$R_1 - mg = ma$

$R_1 - mg = 0$ ($a = 0$).

$R_1 = mg = 100 \times 9.8 = 980$ N

Let R_2 = reaction between the floor and the load at constant acceleration.

$R_2 - 100 \times 9.8 = 100 \times 2$

$R_2 - 980 = 200$

$R_2 = 200 + 980$

$R_2 = 1180$N

CLASS ACTIVITY:

A body of mass 20kg is placed in a lift. Find the reaction between the floor of the lift and the body when the lift moves downward with retardation of 2. $5m/s^{2.}$ (Take g= $10m/s^2$).

A body of mass 15kg is placed on a smooth plane which is inclined at 60^0 to the horizontal. Find (a) the acceleration of the body as it moves down the plane

The velocity that the body attains after 5seconds if (i) it start from rest; (ii) it moves with an initial velocity of 4 ms^{-1} .

SUB TOPIC: MOTION ALONG AN INCLINED PLANE.

DIAGRAM

$F - mg \sin \alpha = ma$, if $F > mg \sin\alpha$ (a = acceleration upward plane)

$Mg \sin\alpha - F = ma$, if $F < mgsin\alpha$ (a = acceleration downward plane)

Example 1. An object whose weight is 20kg is placed on a smooth plane inclined at 60^0 to the horinzontal.

Find: (a) the acceleration of the object as it moves down the plane ; (b) the velocity attained after 6 seconds if : (i) it starts from rest ; (ii) it moves with an initial velocity of $10ms^{-1}$. (Take g= $10ms^{-1}$)

Solution

Net force acting on the body down the plane is mgsin 60^0

The force acting on the body upward is zero.

Hence, mg sin 60^0= 20 x a.

$10 \times 10 \sin 60^0 = 20 \times a$

$100\sqrt{3}/2 = 20\ a$

$50\sqrt{3} = 20\ a$

$a=25\sqrt{3}\ ms^{-2}$

OR $a = 21.7ms^{-1}$

b(i) when the body start from rest u=0

using equation of motion

$v = u + at$

$v = 0 + 10 \times 6 = 60\ ms^{-1}$

b(ii) if the body move with an initial velocity of 10 ms^{-1} , then;

$v = u + at$

$v = 10 + 6 \times 10 = 70\ ms^{-1}$

CLASS ACTIVITY:

A body of mass m is placed on the surface of a smooth plane which is inclined at an angle to the horizontal. A force F whose line of action is parallel to the surface of the inclined plane acts on the body to first prevent it from slipping down the plane. If R is the reaction between the surface of the inclined plane and the body, show that $F = R \tan \theta$

SUB TOPIC: MOTION OF CONNECTED PARTICLES

Example: Two vehicles of mass m_1 and m_2 are connected by an inextensible chord whose mass can be neglected. The vehicle of m_1 has broken down and the vehicle of mass m_2 is towing the vehicle of

mass m_1, with a reactive force of T Newton. The two vehicles are moving with acceleration a ms^{-2}. If T is the tension in the chord and any frictional force is to be neglected, show that

(i) $a = F/ m_1+ m_2$ (ii) $T = m_1f/m_1+m_2$

Solution

Vehicle with mass m_1; $T = m_1 a$ …… (1)

Vehicle with mass m_2 ; $F – T = m_2a$ ……..(2)

Adding (1) and (2)

$F – T + T = m_1a + m_1a$

$F = a (m_1+ m_2)$

$a = f/m_1 + m_2$

(ii) substituting the value of "a" into equation …….. (i)

Therefore $T= m_1 x f/m_1 + m_2 = m_1f/m_1 + m_2$

CLASS ACTIVITY:

Two particle of masses 10kg and 8kg are connected by a light in extensible string which is passed over a light frictionless pulley. Find the tension in the string and the acceleration with which the particles move when released.

Two bodies of masses 2m and 3m are connected by a light in elastic string which is passed over a smooth fixed pulley. The body of mass 2m lies on a smooth horizontal table while the body of mass 3m. hangs freely. If the two bodies move with an acceleration a and T is the tension in the string, find expression for

a in terms of g

T in terms of g and m where g is the acceleration due to gravity.

PRACTICE EXERCISE:

Objectives

1. Any body whose dimensions can be neglected is called a …….

2. …… is the amount of matter contained in a body

3. …… is the product of the mass of the particle and its velocity.

4. Action and reaction are …… and ……..

Essay

A force P acts on a body of mass 5kg on a smooth horizontal floor. If it produces an acceleration of $4.5m/s^2$, find the magnitude of P.

A body of mass 100kg is placed in a lift. Find the reaction between the floor of the lift and the body when the lift moves upward: (i) at constant velocity (ii) with an acceleration of $3.5m/s^2$, (Take g = $10m/s^2$).

A body of mass 50kg is placed in a lift. If the lift moves upward with a retardation of $2m/s^2$,find the reaction between the floor of the lift and the body. (Take g = $9.8m/s^2$).

Two bodies of masses 2m and 3m are connected by o light inelastic string which is passed over a smooth fixed pulley. The body of mass 2m lies on a smooth horizontal table while the body of mass 3m hangs freely. If the two bodies moves with an acceleration a and T is the tension in the string, find the expression for: (i) a in terms of g (ii)T in terms of g and m where g is the acceleration due to gravity.

ASSIGNMENT

State the three law of motions

Derive the formula F = ma

What will be an acceleration of mass of 50kg when acted upon by a force of 30N?

A ball fall from rest down a smooth plane inclined at an angle to the horizontal. How long in seconds, will it take the ball to cover a distance of *d*m along the plane?

A body of mass 2kg moving with the velocity 5m/s due East collides with another body with velocity 4m/s. if the heavier body is brought to rest by the collision, find the velocity of the lighter body after collision.

DYNAMICS 2

CONTENT

(a) Work, Power and Energy: Impulse and momentum.

(b) Projectiles: Trajectory of projectiles; greatest height reached; Time of flight; Range and Projection along inclined plane.

SUB TOPIC: WORK, POWER AND ENERGY: IMPULSE AND MOMENTUM.

Work, Power and Energy

Work: the work done (W) is defined as the product of the force (F) and the displacement or distance (d) must be in the direction of the force.

⇒ Workdone (W) = Force (F) displacement (d). ⇒W = F. The unit of work is Joule (J).

Examples:

How much work is done by a force of 18N that moved a load of 500kg through a displacement of 6m?

Solution:

F = 180N d = 6m ⇒ W = fd = 180(6) = 1080J.

A particle whose mass is 34kg is acted upon by a force. If the particle moves an upward distance of 100m in the line of action of the force, find the workdone. (take g = 10m/s^2 .

Solution: f = ? m = 34kg g = 10m/s^2

F = mg = 34(10) = 340N

W = fd = 340(100) 3400J.

POWER

Power is the rate at which work is being done. The unit watt(W). power is also defined as the rate of transfer of energy.

Power (P)

Example:

A car travelling at 60m/s produces a force of 400N. calculate the power of the engine.

Solution:

F = 400N, V = 60m/s P = Fv P = 400(60) = 24000W.

A force of 20N applied parallel to the horizontal moves a body through a distance of 40cm. what is the workdone by the force?

ENERGY

This is the ability or capacity of a body to do work. The unit of energy is the same as that of work i.e. Joules (J). There are two form of energy;

Kinetic Energy denoted as E_K. Mathematically, E_K ^{2}. Where m = mass and v = velocity.

Potential energy denoted as E_P. By formular E_P = mgh, where m =mass, g = acceleration due to gravity, h = height.

Example:

A body of mass 50kg is raised to a height of 2m above the ground. What is the potential energy?

Solution:

M = 50kg, h = 2m, g = 10m/s^2

E_P = mgh = (50)(10)(2) = 1000J

A body of mass 25kg is raised to a height of 4m above the ground. If the is allowed to fall Calculate its kinetic energy

Just before impact with the ground (g = 10ms^{-2})

Solution:

E_K ^{2} But $v^2 = u^2 + 2as = 0 + 2as = 2as$(body from rest u = 0)

$v^2 = 2as$ But g = a and s = h.

$v^2 = 2gh = (2)(10)(4) = 80$

E_K ^{2} =

IMULSE AND MOMENTUM

Impulse is the time – effect of force. It is expressed as *Fdt = mdv* (where is the impulse while on the mass of the body or particle.

Momentum is the velocity – effect on the mass of the body or particle. It represented by

The time – effect of force is equal to the change in momentum.

The unit of impulse and momentum is Ns(Newton – Second)

Example:

A body at rest and of mass 10kg is acted upon by a force of 40N for 0.4S. find

The increase in momentum
The speed of the body
The distance covered within the time interval

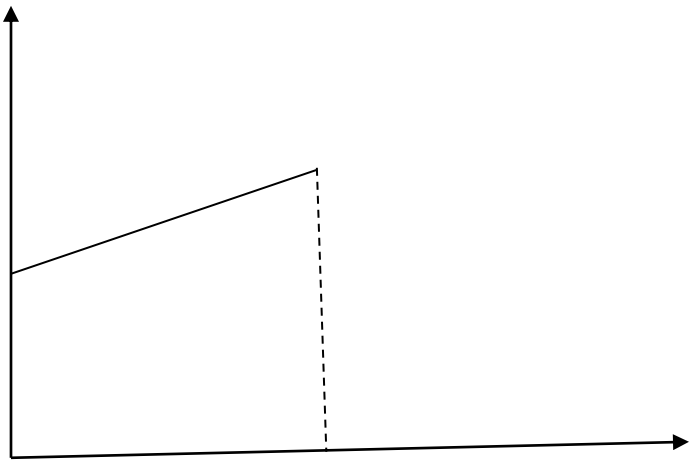
Solution:

F = 40N, m = 10kg. Impulse on the body = 40(0.4) = 16NS
Ft = mv – mu (u = 0), 16 = 10v ⇒ v = 1.6m/s.
V = u + at, 1.6 = 0 + a(0.4) ⇒ a = 4m/s^2
Difference ⇒ S = ut + ½ at^2 = 2(0.16) = 0.32m.

CLASS ACTIVITY:

A body of mass 50kg is placed in a left. If the lift moves upward with a retardation of 2m/s^2, find the reaction between the floor of the lift and the body. (Take g = 9.8m/s^2).
A body at rest and of mass 8kg is acted upon by a force of 30N for 0.4seconds. find the:
Impulse on the body.
Final speed of the body.
Distance covered within the time interval.
When a force of 250N acts on a particle, the particle is displaced a distance along the of action of the force. If the work done in the process is 3250J.
A body is subjected to the action of a force F, such that the force – distance graph of the body is as shown below:

SUB TOPIC: PROJECTILES



Projectiles describe the motion of a particle or body under gravity in two dimensions. When that happens we have the following equations of motion.

$V_x = u\cos$
$V_y = u\sin$
$S_x = ut$
$S_y = ut\sin^2$
$V^2_x = u^2\cos^2$
$V^2y = u^2\sin^2_y$

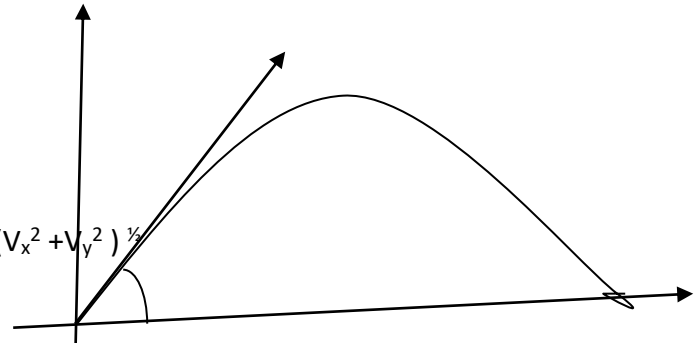
Where: S_x = horizontal distance, S_y = vertical distance, V_x = horizontal component of the velocity, V_y = vertical component of the velocity.

Example:

A particle is projected with an initial velocity of 30m/s at an angle of 70^0 to the horizontal after 2 seconds. Find the:

Vertical component of the velocity
Horizontal component of the velocity
Magnitude of the velocity
Vertical distance travelled
Horizontal distance travelled.

Solution:



$V_y = u\sin$
$V_y = 30\sin70^0 – 9.8$
$V_x = u\cos$
$V_x = 30\cos70^0 = 10.3m/s$
If v = velocity magnitude, Then, $V = (V_x^2 + V_y^2)^{½}$
$V = (8.6^2 + 10.3^2)^{½} = m/s^2$
$S_y = ut\sin^2$
$S_y = 30\text{Sin}70^0 – ½ (9.8)(2^2) = 28.19 – 19.6 = 8.6m$
$S_x = ut = 30^{0} = 20.52m.$

GREATEST HEIGHT REACHED.

The formular for greatest height reached by a body when the vertical component of the velocity is zero is given as:

OR

TIME OF FLIGHT

This is the time it take a projected body to reach it greatest height and returned to it original position.

RANGE

The range is the horizontal distance covered when the particle or body returns to its origin level.

TRAJECTORY OF A PROJECTILE.

This is the path followed or traced by the projected particle. The part is best described as PARABOLA.

Equation: ^{2}

Example:

A particle is projected with an initial speed of 45m/s at an angle of elevation of given by

$\sin^{-1}(0.5)$. find the:

Gretest height reached by the particle
Horizontal range
Time of flight
Magnitude velocity after 2.2s.

Solution:

$^{-1}(0.5) = 30^{0}$;

$H = 45^{22}/2 = 25.83m.$
$S_y = utsin^2$
At the range $S_y = 0 \Rightarrow utsin^2 = 0 \Rightarrow$

Vx =

V_y = usin

Let V = magnitude of the velocity after 2.2s

Then, V = $V_x^2 + V_y^2$ =

CLASS ACTIVITY:

A particle is projected into the air with a speed of 59m/s at an inclination $\sin^{-1}(3/5)$. Find:

Greatest height reached by the particle.

Horizontal range

Time of flight.

PROJECTION ALONG AN INCLINED PLANE.

V'_x

V'_y

Where V'_x =component of the velocity along the horizontal plane V'_y = component of the velocity in a direction perpendicular to the plane.

PRACTICE EXERCISE

A projectile is fired with a velocity of 63m/s and at an angle of elevation of 60^0 to the horizontal. Find the:

Greatest height attained

Horizontal range

Time of fight

Speed of the particle when the projectile is 75m above the horizontal. ($g = 10m/s^2$) .

A particle is projected with a velocity v at an angle x to the horizontal. Find the maximum range of the particle.

A body is projected from a point (0,0) with a speed of 80m/s at an angle 30^0 to the horizontal. Find the height attained in 5s(g = 10m/s)

A projectile launched with a velocity of 30m/s. If it just clears a barrier 40m high at a distance 30m away from the point of projection. Calculate correct to one decimal place, the:

Horizontal range

Maximum height attained($g = 10m/s^2$

ASSIGNMENT

A particle starting from rest falls freely from a height H above the ground. If g is the acceleration due to gravity, show from energy consideration that the velocity 'V' with which the particle strikes the ground is given by the expression:

A ball weighing 500N is dropped from a height of 30m. find the kinetic energy of the ball just before striking the ground.

A body is projected from a point such that the horizontal and vertical components of its velocity are 640m/s and 480m/s respectively. (a) calculate the greatest height attained

above the point of projection. (b) if the time of flight is 96seconds, calculate the horizontal range through the point of projection. (Take g = 10m/s).

A particle is projected from the top of a building 12.6m above the horizontal ground with speed 7m/s at an angle 30^0 above the horizontal. Find the time which elapsed before the particle hits the ground.

A body is projected into the air with speed 40m/s at an angle to the horizontal from a height 20m above the ground. Taking g to be 10m/s^2, find the time taken to hit the ground.